(4)

ARL-AERO-R-176

AR-004-596

AUSTRALIA

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORY

MELBOURNE, VICTORIA

Aerodynamics Report 176

A POTENTIAL THEORY FOR THE STEADY SEPARATED FLOW ABOUT AN AEROFOIL SECTION (U)

by

TON TRAN-CONG

Approved for Public Release

FEBRUARY 1988

AR-004-596

DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
AERONAUTICAL RESEARCH LABORATORY

Aerodynamics Report 176

A POTENTIAL THEORY FOR THE STEADY SEPARATED FLOW ABOUT AN AEROFOIL SECTION (U)

by

TON TRAN-CONG

SUMMARY

An incompressible potential flow theory is used to determine the steady separated flow about an aerofoil. The theory permits a continuous variation from fully-attached (Joukowski) flow to fully-separated (Helmholtz) flow, with the Kutta condition always satisfied at the trailing edge, and with the position of the separation point as an assignable parameter to determine the flow configuration. The method is also applicable to other flows such as that about a flat plate with a rear free-stream flap.

DSTO ARL
FISHERMEN'S BEND

POSTAL ADDRESS: Director, Aeronautical Research Laboratory,
P.O. Box 4331, Melbourne, Victoria, 3001, Australia

CONTENTS

1. Introduction

The incompressible inviscid flow about a flat plate is basic to the theory of aerofoils in aerodynamics. Fully -attached flow around a flat plate may be determined by conformal mapping from the flow around a circular cylinder with the Kutta condition that velocity is finite and the pressure is continuous at the trailing edge of the flat plate. Schmieden [1] proposed a method to determine flat plate flows with separation from the rearward surface forming an open. infinite wake (Another class of solution has also been considered by Schmieden in an earlier paper [2]). His work has been largely ignored in the English literature. It is to be shown here that Schmieden's proposal can be extended into a method which spans the full range of admissible separated potential flows about a flat plate and also about an arbitrary aerofoil section. This provides a new theory which spans the whole range from Helmholtz' theory to Joukowski's theory.

Specifically, an extension and modification of Schmieden's method of determining the flow about a flat plate is presented here. It yields the Joukowski and Helmholtz flows as given in Lamb [3], as particular limiting cases, and permits a continuous variation from one flow configuration to the other. The method is simple but is general enough to allow for the representation of flow about an arbitrary aerofoil section when the section is approximated by a polygon. The problem has not been treated in the comprehensive book by Birkhoff and Zarantonello [4] and is of a more general nature than thin wing theory based on perturbation as given in the book by Woods [5]. By altering the complex velocity function other variant flows can also be modelled, among which is the flow about a flat aerofoil with a rear free-stream flap. This flow was described by Hurley and Ruglen [6], and has been considered more recently by Saffman and Tanveer [7].

Separated flow with a finite wake has recently been a topic of interest, as shown in the numerical studies of Ribaut [8,9] and the references cited there. The merit of the present method is that it requires only a very small amount of computation to arrive at a self-consistent potential flow model. This model may then be used as an initial state for iterative processes such as those of Ribaut. It may also be used as a flow model in its own right, as a reasonable representation of a real physical flow, in a similar way to Pinkerton's model [10] in order to obtain a reduced circulation so that the advantage of the latter model is retained but without incurring the violation of Kutta's condition at the trailing edge of the aerofoil.

The new theory presented here does not give an under-pressure inside the finite wake. This is consistent with the statement in Batchelor's book [11] that no mathematical model of inviscid flow without anomalies has been found to satisfy the under-pressure condition. This feature is also characteristic of the whole set of free-stream flows which have appeared since the original paper by Helmholtz. The theory presented here is proposed as a fairly simple solution upon which more refinements may be added to arrive at some better predictions than are afforded by existing methods; indeed, a more elaborate model with recirculation is under investigation.

It may be noted that the reunion of free streamlines behind an obstacle has been a subject of controversy. Southwell and Vaisey [12] obtained such reunion behind a circular cylinder with their relaxation method. This reunion was subsequently questioned by G.I.Taylor and doubt was raised whether it

was due solely to the use of numerical approximation. The existence of such a reunion point in a symmetrical potential flow was first shown by Lighthill [13] in answer to Taylor's question. The results in this paper confirm the existence of reunion points in more general asymmetrical flows.

2. Flow about a Flat Plate

Using Helmholtz' method, we define a complex velocity potential

$$w = \phi + i\psi \quad (\phi, \psi \text{ real}), \tag{1}$$

and a complex function ζ is introduced

$$\zeta = \ln(\frac{dz}{dw}) = -\ln q + i\theta \tag{2}$$

where q and θ are the magnitude and angle of the velocity vector in the (physical) z-plane. The flow under consideration is shown in the physical and the hodograph planes in figures 1a and 1b. In figure 1a, the physical flow is from the left to the right. The dividing streamline DA comes to a stagnation point at A and then follows two different paths along ABCWD and AED. The upper trailing free stream line CWD leaves the trailing surface of the flat plate tangentially. As this free streamline begins with its concavity initially on its upper side and proceeds to infinity with the concavity finally on its lower side, it must have an inflection point W somewhere between C and D. The inflection point W is not easily seen in figure 1a due to the small degree of curvature and the scale of the plotted figure. The area behind the flat plate, bounded by the curve DWCED is the separation wake with constant pressure inside and with no internal flow. The channel with the breakwater in figure 1b corresponds to the physical flow outside the separation wake of figure 1a. The solution to this flow with an infinite wake is due to Schmieden. The method of solution given below will be slightly different from that of Schmieden's original work so that it can be extended smoothly into flows where the two trailing free stream lines reunite at a finite distance from the aerofoil.

Introducing a σ plane such that

$$\frac{dz}{dw} = C\frac{(\sigma - \bar{\sigma}_A)(\sigma - \sigma_B)}{(\sigma - \sigma_A)(\sigma - \bar{\sigma}_B)}, \tag{3}$$

the flow shown in figure 1c is obtained. The complex scaling constant C determines the size and the orientation of the flow in the z-plane. The upper semi-circle corresponds to the constant direction paths in the z-plane and the real axis corresponds to the free streamlines in the z-plane, which have constant q. The center W of the semi circle is chosen to correspond to the inflection point of the free streamline DABCWD in the z-plane. It corresponds to the tip of the breakwater in the ζ plane, which is the point where there is a reversal in the variation of the direction of the velocity vector along a free streamline in the physical plane. A consequence is that σ_A is equal to $-\bar{\sigma}_B$. The existence of the inflection point W in the infinite wake case is the consequence of the assumption that the upper trailing free stream line leaves the trailing surface of the flat plate tangentially. Here it is assumed that exactly one inflection point exists on one of the two trailing free stream lines. This assumption will be justified *a posteriori*. The method

of solution here differs from that of Schmieden in that the inflection point W is chosen to be the center of the semicircle of the σ plane rather than D being the center of the semicircle of Schmieden's τ plane.

Putting D on the real axis of the semi-circle and taking the complex function $w(\sigma)$ to be

$$w(\sigma) = \frac{1}{(\sigma - \sigma_D)^2} + \frac{\sigma^2}{(1 - \bar{\sigma}_D \sigma)^2} - \frac{4\mu}{(\sigma - \sigma_D)} - \frac{4\mu\sigma}{(1 - \sigma_D \sigma)} \tag{4a}$$

so that the imaginary part of $w(\sigma)$ is constant along the boundary of the semi-circular disc of the σ plane, with μ being a real constant gives Schmieden's solution to the flow with z determined by

$$z = \int e^{\zeta} \frac{dw}{d\sigma} d\sigma. \tag{5}$$

As D is put at various positions from 1 to 0 on the real axis of the σ plane the flow varies continuously from Helmholtz' fully separated flow to flow with a wake closing at infinity. In the limit when D is close to 1 in the σ plane, the flow tends to the configuration of Helmholtz's flow. The separation point C is then very close to the leading edge B of the flat plate and the inflection point W is also very close to B. Therefore the upper free streamline appears as if it was leaving the leading surface of the flat plate at the point B as in Helmholtz's result. The point D cannot be put on the negative part of the real axis of the σ plane as this creates free streamlines which cross at a finite distance from the trailing edge. The latter kind of solution is unacceptable. The above result is essentially the same as given in Schmieden's paper except for the choice of the inflection point W as the center of the semicircle. It is this choice which allows the smooth transition to a flow with a finite wake.

It is found here that as the point D cannot be put to the left of the center of the semi-circle it can be put above the real axis, having its associated complex potential $w(\sigma)$ as

$$\begin{aligned} w(\sigma) = {} & e^{i\beta}\left(\frac{1}{\sigma - \sigma_D}\right) + e^{i\beta}\left(\frac{\sigma}{1 - \sigma_D \sigma}\right) + e^{-i\beta}\left(\frac{1}{\sigma - \bar{\sigma}_D}\right) + \\ & + e^{-i\beta}\left(\frac{\sigma}{1 - \bar{\sigma}_D \sigma}\right) + i\gamma \ln \frac{(\sigma - \sigma_D)(1 - \sigma_D \sigma)}{(\sigma - \bar{\sigma}_D)(1 - \bar{\sigma}_D \sigma)}, \end{aligned} \tag{4b}$$

which is chosen so that the imaginary part of $w(\sigma)$ is constant along the boundary of the semicircular disc of the σ plane, with β,γ being real numbers corresponding to a doublet plus a vortex at σ_D and its three image points. A closed wake flow is then obtained. The two trailing free streamlines close at the point z_F in the physical plane, which corresponds to the point σ_F on the negative real axis of the semi-circle in the σ plane. The real numbers β and γ are determined by the following two equations

$$\gamma = \frac{\mathrm{Im}\left(e^{i\beta}\left[\frac{\sigma_A}{(\sigma_A - \sigma_D)^2} - \frac{\bar{\sigma}_A}{(\bar{\sigma}_A - \sigma_D)^2}\right]\right)}{\mathrm{Re}\left(\frac{\sigma_A}{\sigma_A - \sigma_D} - \frac{\bar{\sigma}_A}{\bar{\sigma}_A - \sigma_D}\right)} \tag{6}$$

and

$$\frac{i\gamma}{e^{i\beta}} = \left\{\frac{d}{d\sigma}\left[\ln \frac{dz}{dw}\right]\right\}_{\sigma = \sigma_D} \tag{7}$$

which require that A be a stagnation point and that the integral given by formula (5) vanish along the contour ABCFEA respectively; these two equations will be discussed in detail later.

As the point σ_D moves upwards leaving the center while tending toward the top of the semi-circle, the flow in the physical z-plane closes nearer and nearer to the trailing edge of the flat plate. In the limit as σ_D tends to $+i$ with σ_A and σ_B moving towards the same point to keep the angle of attack constant, the flow in the z plane tends to the Joukowski flow about a flat plate with its angle of attack α determined by the relative positions of σ_D, σ_A and σ_B. The value of γ represents the circulation about the geometry formed by the aerofoil and its closed wake. As the flow tends towards the Joukowski flow the circulation γ increases toward that required in Joukowski flow. Hence a flow with a closed wake may be regarded as a "reduced circulation" flow.

Consider now the two equations (6) and (7). To satisfy them both, the point σ_D must follow a specific curve joining $\sigma = 0$ and $\sigma = i$ for each given value of angle of attack α (i.e. each angle of attack α corresponds to a different curve), as illustrated by the lines of constant α in figure 5. It is seen that for each given σ_A and $\sigma_B = -\sigma_A$, the difference $h(\sigma_D)$ between the two values of γ given by equations (6) and (7),

$$h(\sigma_D) = \frac{\operatorname{Im}\left(e^{i\beta}\left[\frac{\sigma_A}{(\sigma_A-\sigma_D)^2} - \frac{\bar{\sigma}_A}{(\bar{\sigma}_A-\sigma_D)^2}\right]\right)}{\operatorname{Re}\left(\frac{\sigma_A}{\sigma_A-\sigma_D} - \frac{\bar{\sigma}_A}{\bar{\sigma}_A-\sigma_D}\right)} + 2ie^{i\beta}\sigma_D\left[\frac{1}{\sigma_D{}^2-\bar{\sigma}_A^2} - \frac{1}{\sigma_D{}^2-\sigma_A{}^2}\right]$$

tends to $+\infty$ and $-\infty$ when the argument of σ_D tends to 0 and to π respectively while the magnitude of $0 < |\sigma_D| < 1$ is kept constant. The reason is that the last term remains finite and the first term is approximated by $-\cos\beta/\operatorname{Im}(\sigma_D)$. Since $h(\sigma_D)$ is continuous inside the semi-circular disc there must be a zero of $h(\sigma_D)$ for each value of $|\sigma_D|$. Also for each value of $\sigma_A = -\bar{\sigma}_B$, the argument $\angle\sigma_D$ of σ_D required such that $h(\sigma_D)$ vanishes is a continuous function of the corresponding magnitude $|\sigma_D|$. Since $h(\sigma_D)$ is positive for $\angle\sigma_D$ close to 0 and negative for $\angle\sigma_D$ close to π and $h(\sigma_D)$ is continuous it is obvious that there must be a continuous line joining $\sigma_D = 0$ and $|\sigma_D| = 1$ on which $h(\sigma_D)$ is zero. As the angle of attack is a continuous function of σ_D and σ_A there is a continuous line joining $\sigma = 0$, $\sigma = i$ for each angle of attack smaller than $\pi/2$ such that the function $h(\sigma_D)$ is zero on this line. This is depicted in figure 5.

It is noted that this proof of existence of the locus for σ_D such that $h(\sigma_D) = 0$ holds irrespective of the form of the function dz/dw as long as we have

$$\lim_{\sigma_D\to(-r)}(\beta)\cdot\lim_{\sigma_D\to r}(\beta) < 0,$$

with $\beta(\sigma_D)$ determined from equation (7) only.

All solutions for separated flow about a flat plate are given in the graph of figure 5. This graph relates the values of Argument(σ_A), σ_D, separation distance s and angle of attack α.

Using the results given in the graph, the coefficients of lift, drag and moment are plotted for each value of angle of attack α and the distance s from the leading edge to the separation point as in figures 6a, 6b and 6c.

Inspection of figures 2, 1, 3 and 4 shows that as the separation point on the upper surface of the flat plate moves rearwards the flow closes and the reunion point moves such that the flow approaches the Joukowski configuration. In full Helmholtz flow the leading edge B, the separation point C and

the inflection point W coincide, the flow appears as if having its upper free streamline leaving its leading surface tangentially at the leading edge. As the two trailing streamlines tend to close up, the inflection point W moves away from the separation point C, tending towards infinity. After closure of the two free streamlines, the inflection point W moves into the lower trailing streamline and approaches the trailing edge E as the flow tends towards a Joukowski flow. Our assumption that there is exactly one inflection point W on the two trailing free streamlines has been justified since the resulting flows form admissible solutions to the problem.

At any angle of attack of the flat plate, the drag value can range between a maximum value corresponding to Helmholtz flow and zero as in a closed wake flow. It is noted that the lift in this theory can still be positive for angle of attack higher than $\pi/2$, provided the separation point is appropriately chosen as illustrated in figure 6a. As can be seen in figures 5 and 1c, when the point D is not on the positive part of the real axis of the σ plane, i.e. in finite wake flows, it is on the left hand side of the semi-circular disc on this plane, which corresponds to the left hand side of the breakwater of figure 1b. This makes the flow speed on the trailing free stream lines lower than the flow speed at infinity. By Bernoulli's theorem, the pressure on these trailing free streamlines, and hence the pressure inside the finite wake, must be higher than the pressure at infinity. This is not found to occur experimentally, and is a common shortcoming of freestream flow models as already discussed in the introduction part of the paper.

3. Flow about an arbitrary aerofoil

The existence of separated flow about an arbitrary aerofoil is now established. The result was conjectured by Schmieden using only physical reasoning.

First using equation (1) and (2) the potential w is established as in the previous section. The physical z-plane is as in figure 7 and a χ-plane defined by

$$e^{\zeta} = C\left(\frac{\chi^2 - \chi_B^2}{\chi^2 - \chi_A^2}\right),$$

with C being a complex scaling constant. Obviously the aerofoil boundary in the χ plane is not a semi-circle of unit radius but is distorted as depicted in figure 8.

In accordance with Riemann's mapping theorem, the area bounded by the curve ACWDEA can be mapped onto the upper semi-circular section in the σ plane of figure 1c such that $\sigma_W = 0$, $|\sigma_A| = |\sigma_C| = |\sigma_E| = 1$ and $d\chi/d\sigma > 0$ at $\chi = 0$. In this way the transformation $\chi(\sigma)$ is given by

$$\chi(\sigma) = \sum_{n=1}^{\infty} b_n \sigma^n \tag{8}$$

where b_n are the real valued cofficients of the series and $b_1 > 0$. The series so defined is convergent for all $|\sigma| \leq 1$.

The results presented above cover Schmieden's conjecture: The separated flow around any given aerofoil corresponds to a separated flow around a flat plate. The correspondence is given by equation (8).

The composite function $\zeta(\chi(\sigma))$ for this flow is thus defined for all σ inside the upper semi-circular disc. By writing

$$\zeta(\chi(\sigma)) = \ln\left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A}\right) + i\phi(\chi(\sigma))$$

it can be proved that the function $\phi(\chi(\sigma))$, which is

$$i\phi(\chi(\sigma)) = \ln\left(\frac{\chi - \bar{\chi}_A}{\chi - \chi_A}\right) - \ln\left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A}\right) + \ln\left(\frac{\chi - \bar{\chi}_B}{\chi - \chi_B}\right) + \text{constant},$$

is analytic on the semi-circular disc and that $\phi(\chi(\sigma))$ is real for $-1 \leq \sigma \leq 1$. Therefore $\zeta(\chi(\sigma))$ takes the form

$$\zeta(\chi(\sigma)) = \ln\left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A}\right) + i\sum_{n=1}^{\infty} n a_n \sigma^n + i a_0, \tag{9}$$

with all $a_1, a_2, a_3, \ldots$ real. The constant a_0 is real for real σ_D and is complex otherwise. The condition for the stagnation point to be at A remains the same as for a flat plate and is given by equation (6). The closure condition (7) now becomes

$$\frac{i\gamma}{e^{i\beta}} = \left[\frac{d\zeta}{d\sigma}\right]_{\sigma=\sigma_D}$$

or

$$\frac{i\gamma}{e^{i\beta}} = \left[\frac{1}{\sigma_D - \bar{\sigma}_A} - \frac{1}{\sigma_D - \sigma_A} + i\sum_{n=1}^{\infty} n^2 a_n \sigma_D^{n-1}\right]. \tag{10}$$

When σ_D sweeps a semi-circular arc described by $\sigma_D = re^{i\theta}$ ($0 < r < 1$, r kept constant, θ increasing from 0 to π) the first two terms being $-2i\mathrm{Im}(\sigma_A)/[(\sigma_D - \sigma_A)(\sigma_D - \bar{\sigma}_A)]$ do not contribute to the increase in β but the third term may alter the value of β. The right-hand side of equation (10) is the product of $d\zeta/d\chi$ and $d\chi/d\sigma$. Since $\chi(\sigma)$ is a one-to-one and onto conformal mapping, the derivative $d\chi/d\sigma$ cannot vanish anywhere on the disc $|\sigma| < 1$. On the other hand, the derivative

$$\frac{d\zeta}{d\chi} = 2\chi\left(\frac{1}{\chi^2 - \chi_B^2} - \frac{1}{\chi^2 - \chi_A^2}\right) = 2\chi\frac{\chi_B^2 - \chi_A^2}{(\chi^2 - \chi_B^2)(\chi^2 - \chi_A^2)}$$

changes sign once for χ travelling on the real segment (χ_C, χ_E). Therefore β changes its value by π when σ_D sweeps the semi-circular arc described by $\sigma_D = re^{i\theta}$ ($0 < r < 1$, r kept constant, θ increasing from 0 to π). Thus a proof similar to that of the preceding section can be constructed to prove that there exists a continuous line for σ_D such that the pair of equations (6) and (7) are satisfied on that line. Hence with any arbitrary aerofoil at a given angle of attack, the flow can vary continuously from fully attached (Joukowski) flow to fully separated (Helmholtz) flow. It is worthwhile noting that the curvature k for a free streamline of speed q_f is given by

$$k = \frac{d\theta}{|dz|} = -iq_f\left[\frac{d\zeta}{d\sigma}\frac{d\sigma}{dw}\right] \quad \text{for real } \sigma. \tag{11}$$

Hence

$$k = q_f\left[-2\mathrm{Im}\left(\frac{1}{\sigma - \sigma_A}\right) + \sum_{n=1}^{\infty} n^2 a_n \sigma^{n-1}\right]\frac{d\chi}{d\sigma}\frac{d\sigma}{dw} \quad \text{for real } \sigma.$$

It is noted that the curvature is infinite at the starting points and also at the reunion point of both free streamlines. The reason is that $dw/d\zeta$ vanishes at the points C, E and F.

As σ_D tends to 0, σ_A tends to $ie^{-\alpha/2}$ and k tends to 0. Therefore the quantity inside the accolades must vanish. Hence

$$a_1 = 2\cos(\frac{\alpha}{2}) \tag{12}$$

for a flow with free streamlines closing at infinity.

For a flow with finite wake the circulation around the wing is $2\pi\gamma$, where γ is given by the system of equations (6) and (7). The free-stream velocity (at the point D) is given by

$$u_\infty - iv_\infty = (\frac{dw}{dz})_D = \frac{1}{C}(\frac{\lambda_D^2 - \lambda_A^2}{\lambda_D^2 - \lambda_B^2})$$

and the pure lift (there is no drag in flows with finite wakes) is given by

$$\rho 2\pi\gamma \left| \frac{1}{C} \frac{\lambda_D^2 - \lambda_A^2}{\lambda_D^2 - \lambda_B^2} \right| ,$$

where ρ is the density of the fluid. As the two streamlines open the pure lift changes smoothly into lift and drag caused by a stagnation zone behind the aerofoil.

For the practical computation of flow about a given aerofoil, it is convenient to use the following formula

$$\zeta(\sigma) = \ln(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A}) + \int_0^1 \ln(\frac{\sigma - e^{it}}{\sigma - e^{-it}})df(t) + \text{complex constant}$$

where $f(t)$ is a function with bounded variation defined for all real t with $0 \leq t \leq 1$, then each flow regime corresponds to a function $f(t)$. However the construction of such a function $f(t)$ from a given aerofoil shape and selected σ_D is complicated as it involves the determination of the function $f(t)$ to fit a given curve. Figure 9 gives a closed wake flow about an approximated NACA-23012 aerofoil obtained with this method. The aerofoil is represented by a polygon which then gives the function $f(t)$ as a step function. The steps of this function are determined by a Newton-Raphson iteration method.

The reader is also refered to the book by Birkhoff and Zarantonello for other methods of determining flows around a curved obstacle.

4. Flat Plate with rear Free-stream Flap

By making some minor changes to dz/dw different flows can be constructed from the same basic σ -plane. One such variation considered here is the flow about a flat plate aerofoil with a rear free-stream flap. The boundary of this flow is either of constant direction or of constant pressure. The flow is obtained with the selection of dz/dw as

$$\frac{dz}{dw} = C\left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A}\right)\left(\frac{\sigma - \bar{\sigma}_G}{\sigma - \sigma_G}\right)^m$$

where $-m\pi$ is the trailing edge angle of the aerofoil. The flow so obtained is given in figure 10. This result was obtained previously by Hurley and Ruglen (using a different technique) and recently recalculated by Saffman and Tanveer (using yet another technique). Comparing this flow with the separated flow with reunion about a flat plate it can be seen that there are three equations connecting β and γ. This redundant system of equations imposes certain relationships between σ_A, σ_G and σ_D and consequently between z_C, z_G and z_F, as previously suggested by the above four authors. It is noted that the closing condition for this flow is still equation (7) as in the previous two sections.

5. Conclusions

The incompressible, inviscid flow about a flat plate with an assigned separation point on the rearward surface has been described using a single σ-plane. Some of the interesting points about the flow are:

a. For every angle of attack, the flow about a flat plate can vary continuously from fully-attached (Joukowski) flow to fully-separated (Helmholtz) flow, with the Kutta condition always satisfied at the trailing edge, and with the position of the separation point as an assignable parameter to determine the flow configuration. Reduced circulation flow, in which the free streamlines close at a finite distance, and Schmieden's flow are intermediate states.

b. Lift and drag vary continuously as the separation point moves on the rearward surface of the flat plate. Drag is zero for all flows with a finite wake. Lift can remain positive even for angle of attack greater than $\pi/2$ provided the separation point is appropriately chosen. Pitching moment about the quarter chord point can increase or decrease with the angle of attack depending on the chosen position of the separation point.

c. There is only one inflection point on the trailing free streamlines. This inflection point is on the upper free streamline when the flow is open and is on the lower one when the flow is closed.

d. Results analogous to a. and b. also hold for an arbitrary aerofoil section.

Acknowledgement

The author wishes to thank Mr. C.A.Martin for the provision of the reference by Schmieden and the initial computation of flows about a flat plate.

References

1. Schmieden,C.: Flow around wings accompanied by separation of vortices. NACA-TM 961 translation, Luftfahrtforschung 17, No.2 (1940) 37-46.

2. Schmieden,C.: Uber die Eindeutigkeit der Losungen in der Theorie der unstetigen Stromungen. Ing. Arch. 3 (1932) 356-370.

3. Lamb,H.: Hydrodynamics. New York: Dover Publications (1945).

4. Birkhoff,G. and Zarantonello,E.H.:Jets, Wakes, and Cavities. New York: Academic Press (1957).

5. Woods,L.C.: The Theory of Subsonic Plane Flow. Cambridge: Cambridge University Press (1961).

6. Hurley,D.G. and Ruglen,N.: The use of boundary layer control to establish free streamline flows being an introduction to the free streamline flap. ARL-Report A109. Australia (1958).

7. Saffman,P.G. and Tanveer,S.: Vortex Induced Lift on Two Dimensional Low Speed Wings. Stud. in Appl. Maths. (1984) 65-78.

8. Ribaut,M.: A Kinematical Study of Kirchoff-Rayleigh Flow. Z. fur Ang. Math. und Phys. 31 (1980) 83-93.

9. Ribaut,M.: A Vortex Sheet Method for Calculating Separated Two-Dimensional Flows. AIAA J. 21, No. 8 (1983) 1079-1084.

10. Pinkerton,R.M.:Calculated and Measured Pressure Distributions over the Midspan Section of the NACA 4412 Airfoil. NACA Report 563 (1936).

11. Batchelor,G.K.:An Introduction to Fluid Dynamics. Cambridge: Cambridge University Press (1967).

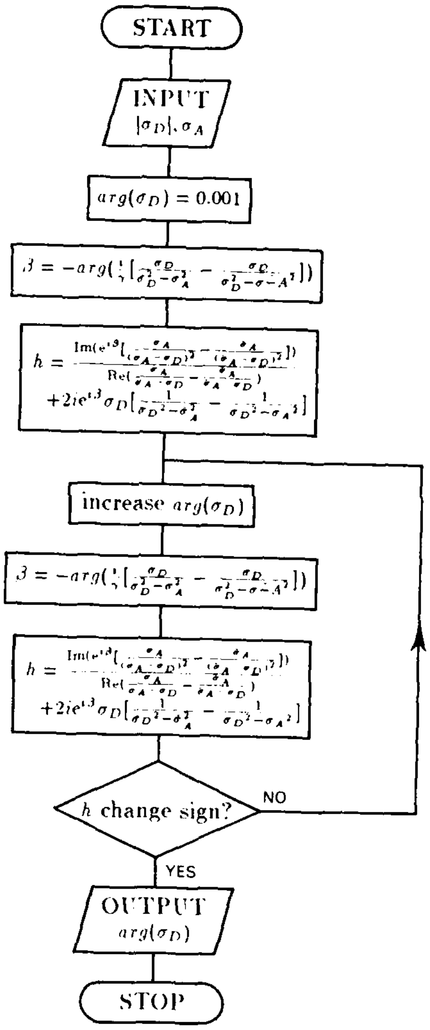
12. Southwell,R.V. and Vaisey,G.: Relaxation Method applied to Engineering Problems. Phil. Trans. R. Soc. Lond. A240 (1946) 117-160.

13. Lighthill,M.J.: A Note on Cusped Cavities. R & M No. 2328 (1945).

APPENDIX A

A fail-proof algorithm for obtaining arg(σ_D) in Flows about a Flat Plate

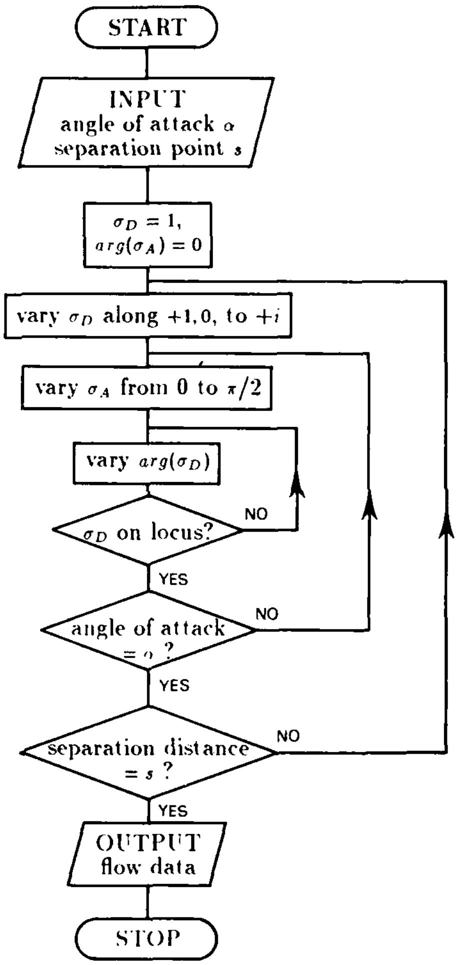
To calculate the value of $arg(\sigma_D)$ when $|\sigma_D|$ is given for flows about a flat plate with reunion of free stream line the following flow chart can be used



APPENDIX B

Algorithm for the calculation of Flow about a Flat Plate

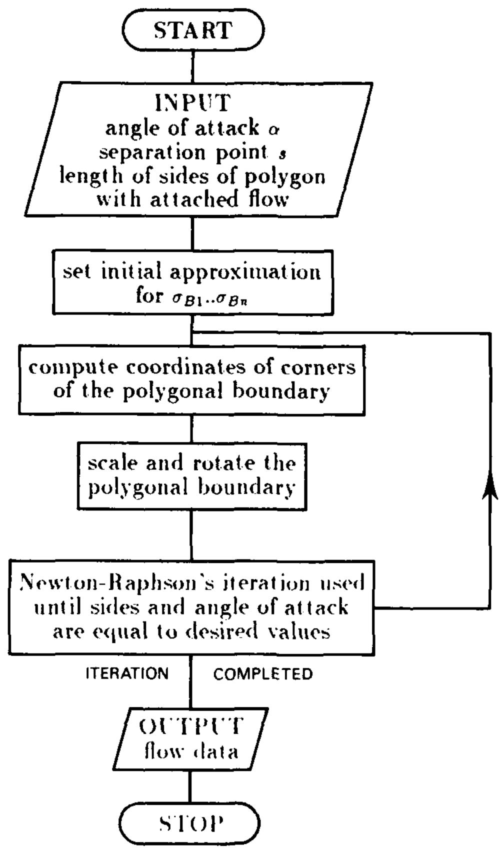
To calculate the flow about a flat plate with given values of α and s the following flow chart is used



APPENDIX C

Algorithm for the calculation of Flow about an approximation (polygonal) wing

To calculate the flow about a wing with a given polygonal cross section (approximating a smooth cross-section) for given values of α and s the following flow chart is used



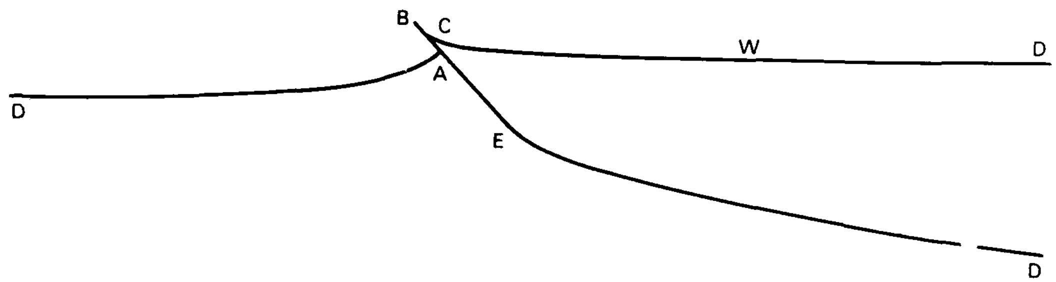


FIG. 1a: SEPARATED FLOW ABOUT A FLAT PLATE IN THE z PLANE. (ANGLE OF ATTACK $=\pi/4$).

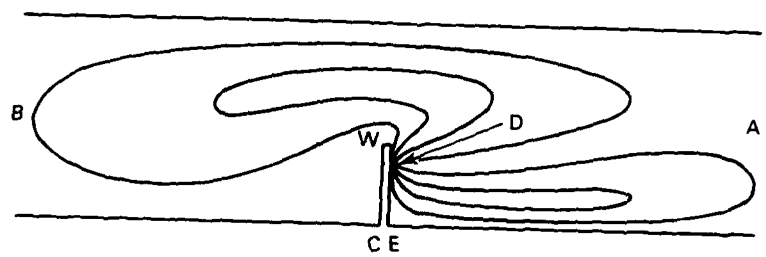


FIG. 1b: SKETCH OF SEPARATED FLOW ABOUT A FLAT PLATE REPRESENTED IN THE ζ PLANE.

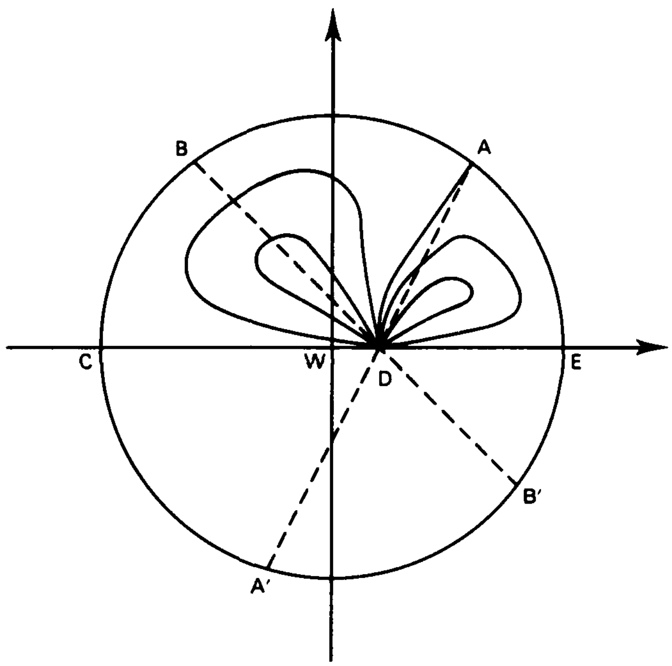


FIG. 1c: SKETCH OF SEPARATED FLOW ABOUT A FLAT PLATE REPRESENTED IN THE σ PLANE.

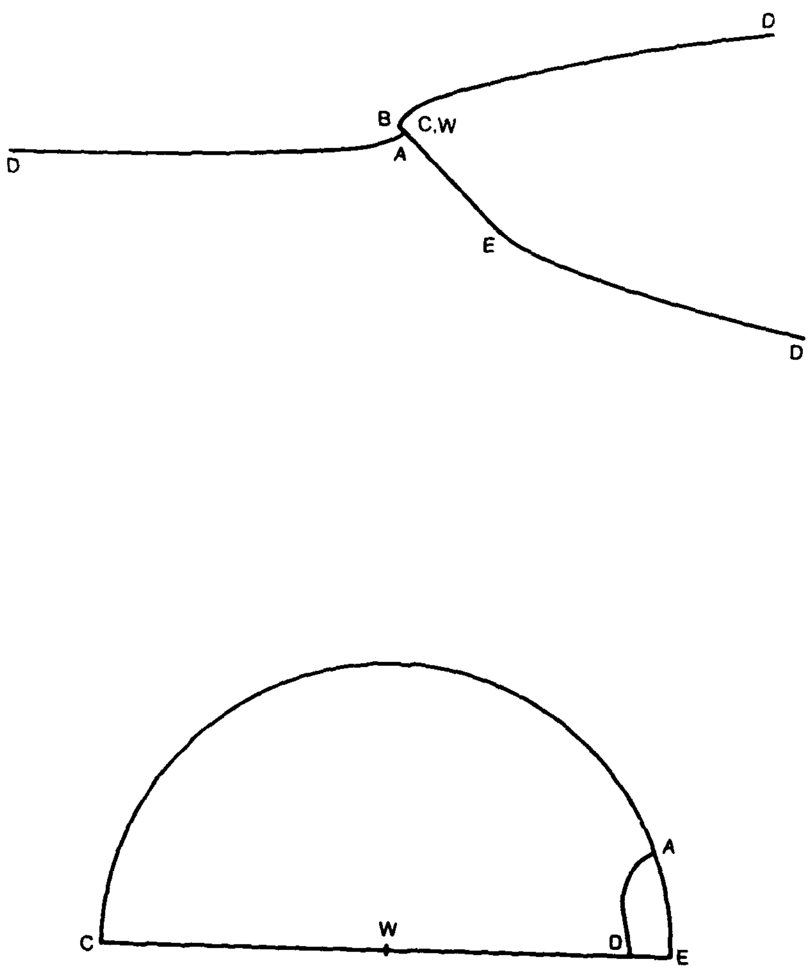


FIG. 2: SEPARATED FLOW ABOUT A FLAT PLATE, WITH $\sigma_D = \sigma_E$ (ANGLE OF ATTACK $= \pi/4$).

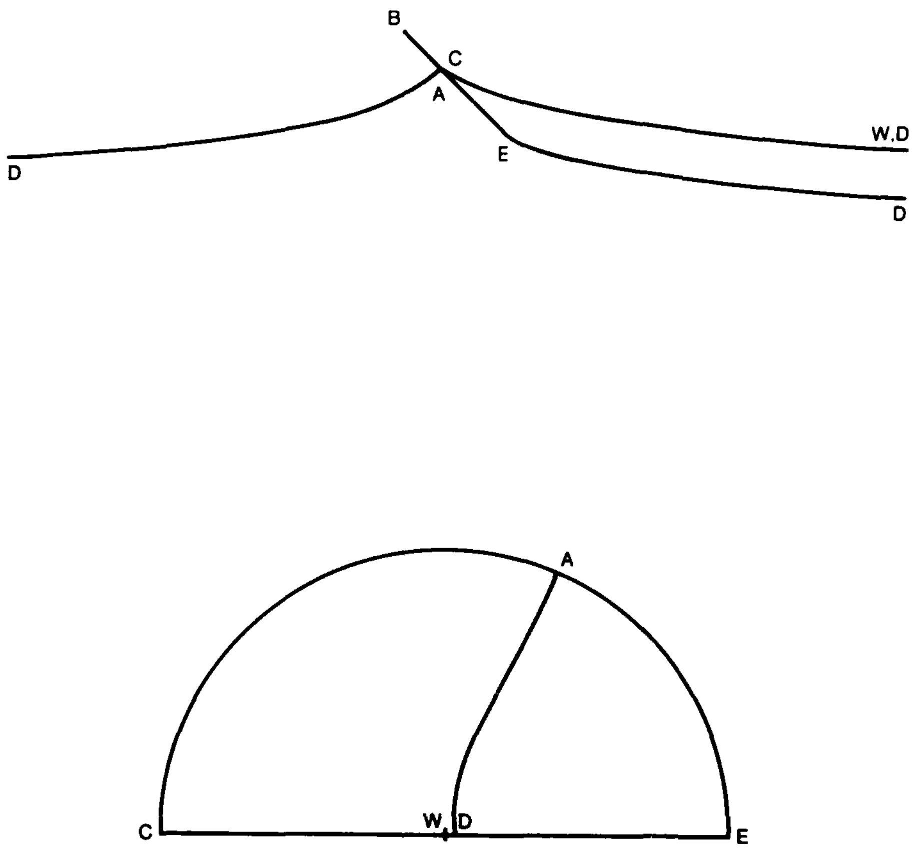


FIG. 3: SEPARATED FLOW ABOUT A FLAT PLATE, WITH $\sigma_D \approx \sigma_W$ (ANGLE OF ATTACK $= \pi/4$).

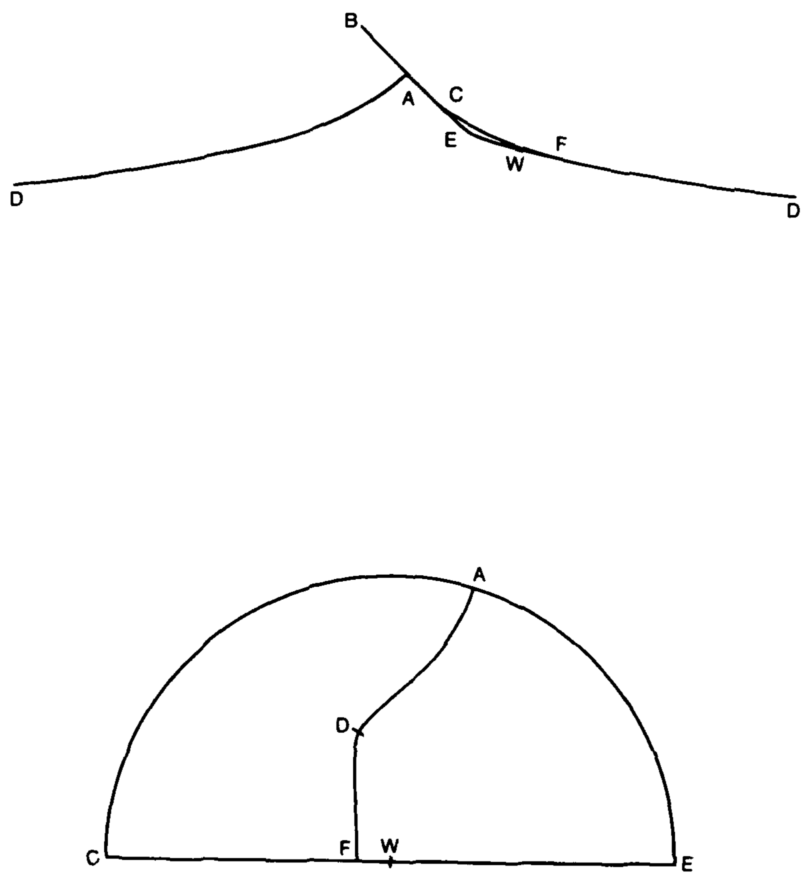


FIG. 4: FLOW ABOUT A FLAT PLATE WITH A FINITE TRAILING WAKE (ANGLE OF ATTACK = $\pi/4$).

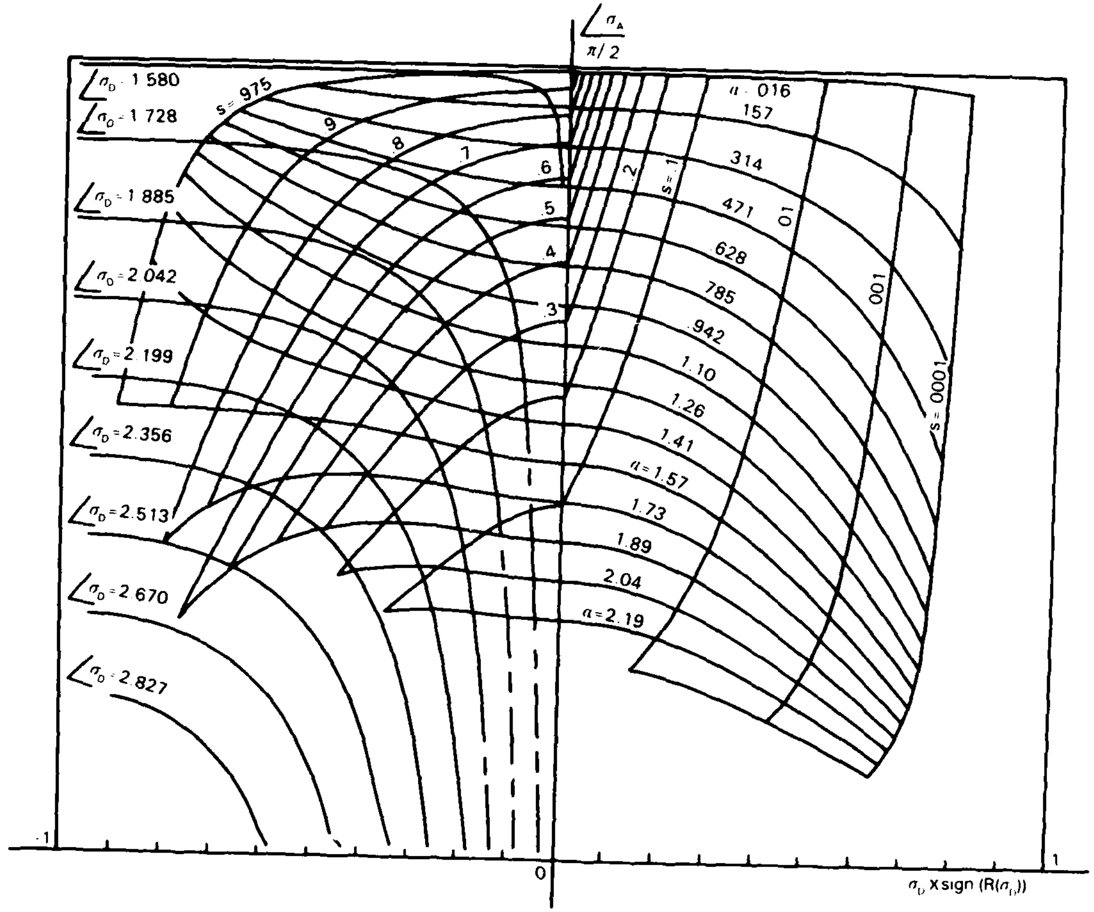


FIG. 5: SOLUTION TO THE FREE-STREAM FLOW ABOUT A FLAT PLATE.

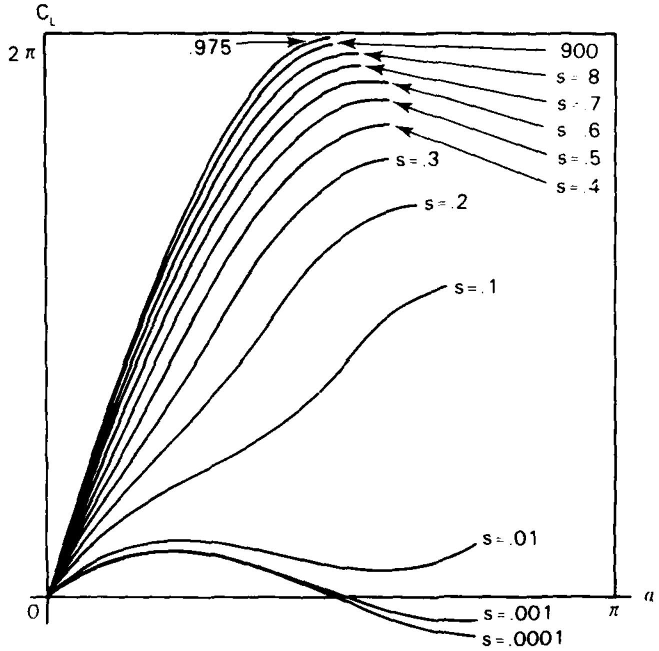


FIG 6a: LIFT COEFFICIENT OF A FLAT PLATE FOR DIFFERENT ANGLES OF ATTACKS AND POSITIONS OF SEPARATION POINT.

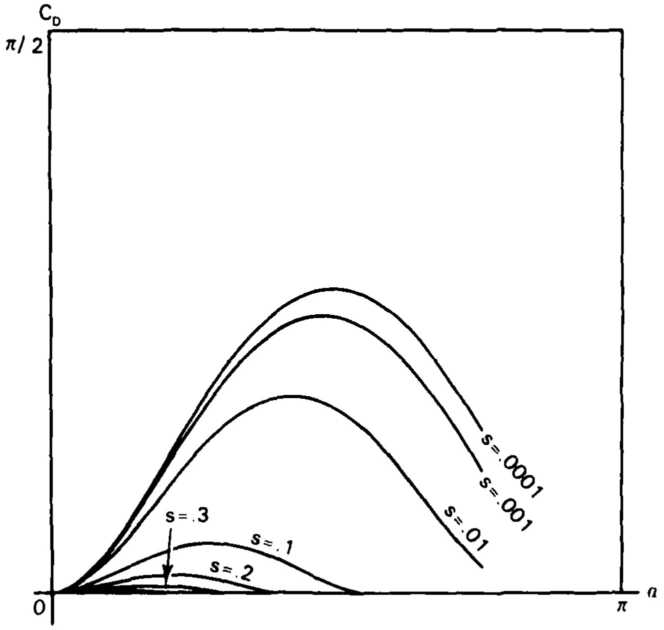


FIG. 6b: DRAG COEFFICIENT OF A FLAT PLATE FOR DIFFERENT ANGLES OF ATTACKS AND POSITIONS OF SEPARATION POINT.

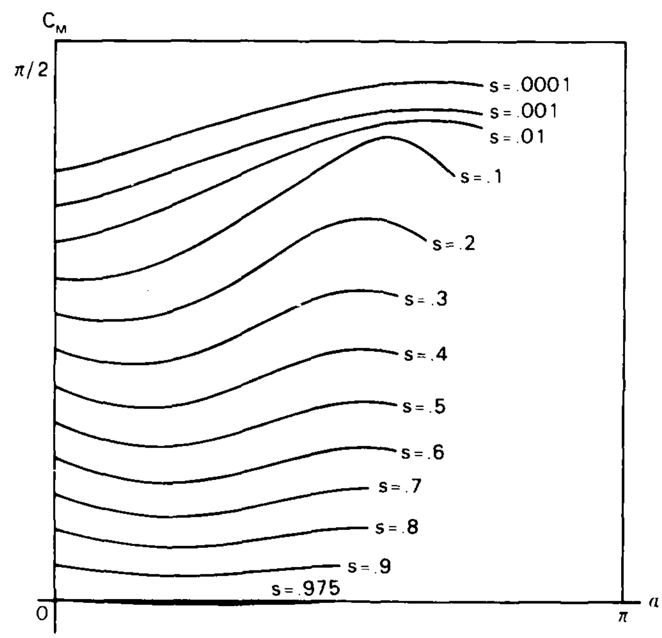


FIG. 6c: MOMENT COEFFICIENT OF A FLAT PLATE FOR DIFFERENT ANGLES OF ATTACK AND POSITIONS OF SEPARATION POINT (SHIFTED CO-ORDINATES USED FOR s = 0.9 to s = 0.0001 WITH EACH CURVES BEGINNING FROM ITS ZERO CO-ORDINATE).

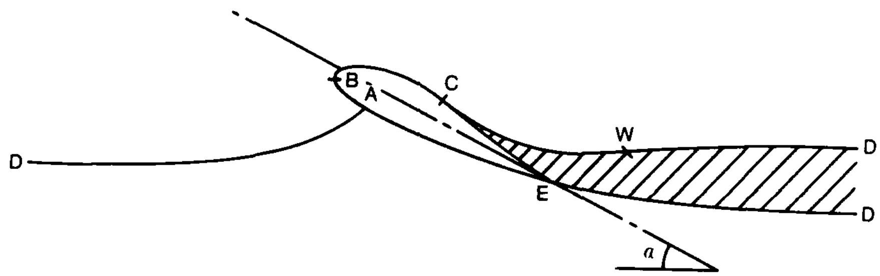


FIG. 7: SKETCH OF SEPARATED FLOW ABOUT AN ARBITRARY AEROFOIL.

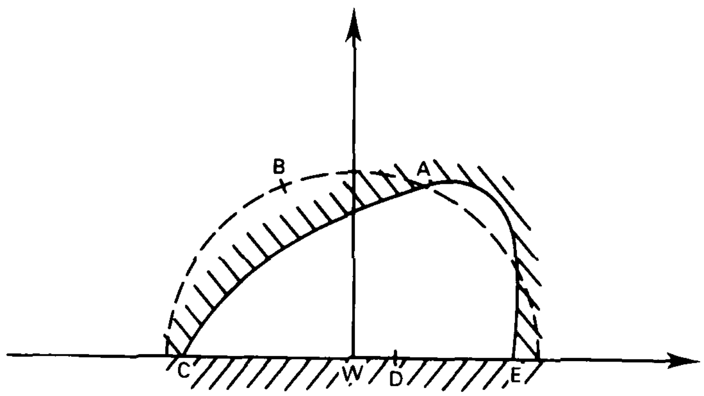


FIG. 8: SKETCH OF SEPARATED FLOW ABOUT AN ARBITRARY AEROFOIL, REPRESENTED IN THE λ PLANE.

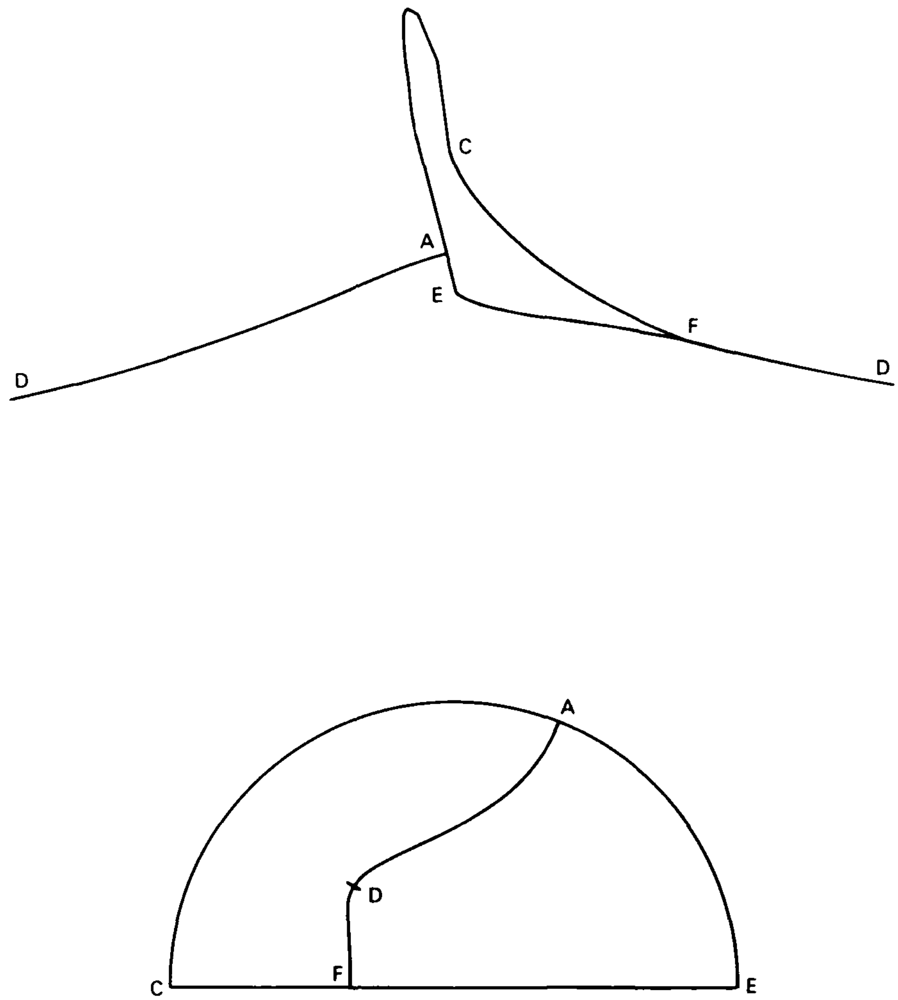
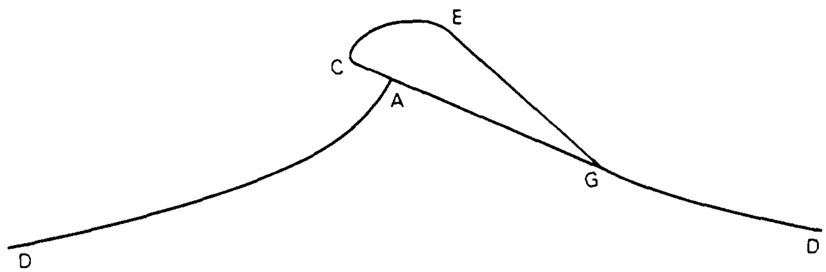


FIG. 9: SEPARATED FLOW ABOUT AN APPROXIMATE NACA 23012 AEROFOIL.



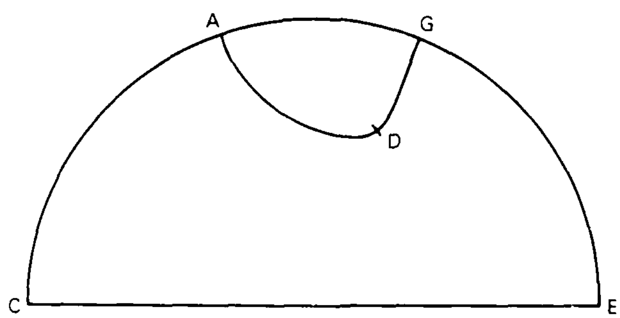


FIG 10. FLOW ABOUT A FLAT PLATE WITH REAR FREE STREAM FLAP, REPRESENTED IN THE z-PLANE AND σ PLANE.

DISTRIBUTION

AUSTRALIA

Department of Defence

Defence Central
- Chief Defence Scientist
- Assist Chief Defence Scientist, Operations (shared copy)
- Assist Chief Defence Scientist, Policy (shared copy)
- Director, Departmental Publications
- Counsellor, Defence Science (London) (Doc Data sheet only)
- Counsellor, Defence Science (Washington) (Doc Data sheet only)
- S.A. to Thailand Military R and D (Doc Data sheet only)
- S.A. to DRC (Kuala Lumpur) (Doc Data sheet only)
- OIC TRS, Defence Central Library
- Document Exchange Centre, DISB (18 copies)
- Joint Intelligence Organisation
- Librarian H Block, Victoria Barracks, Melbourne
- Director General - Army Development (NSO) (4 copies)

Aeronautical Research Laboratory
- Director
- Library
- Superintendent - Aerodynamics and Aero Propulsion
- Head - Aerodynamics Branch
- Divisional File - Aerodynamics and Aero Propulsion
- Author: Ton Tran-Cong
- C. Martin
- M. Cooper
- J.S. Drobik
- P. Gibbens

Materials Research Laboratory
- Director/Library

Defence Science & Technology Organisation - Salisbury
- Library

WSRL
- Maritime Systems Division (Sydney)

Navy Office
- Navy Scientific Adviser (3 copies Doc Data sheet)
- Aircraft Maintenance and Flight Trials Unit
- Director of Naval Aircraft Engineering
- Director of Naval Air Warfare
- Superintendent, Aircraft Maintenance and Repair

Army Office
- Scientific Adviser - Army (Doc Data sheet only)
- Engineering Development Establishment, Library
- Royal Military College Library

Air Force Office
- Air Force Scientific Adviser (Doc Data sheet only)
- Aircraft Research and Development Unit
  - Scientific Flight Group
  - Library
- Technical Division Library
- Director General Aircraft Engineering - Air Force
- Director General Operational Requirements - Air Force
- HQ Operational Command (SMAINTSO)
- HQ Support Command (SLENGO)

Department of Administrative Services
- Bureau of Meteorology, Library

Department of Transport & Communication
- Library
- Flight Standards Division

Statutory and State Authorities and Industry
- Aerospace Technologies Australia P/L, - Manager/Library (2 copies)

Universities and Colleges
- Sydney
  - Prof G.P. Steven, Head, Department of Aero. Engineering
- NSW
  - Professor V.T. Buchwald, Dean, Faculty of Science
  - Library, Australian Defence Force Academy
- Wollongong
  - Professor J.R. Blake, Dean, Faculty of Mathematical Sciences
  - Dr W.K. Soh, Department of Mechanical Engineering
- RMIT
  - Library

FRANCE
- ONERA, Library

GERMANY
- DFVLR Braunschweig, Library

JAPAN
- National Aerospace Laboratory
- National Research Institute for Metals, Fatigue Testing Div.
- Institute of Space and Astronautical Science, Library

Universities
- Kagawa University
  - Professor H. Ishikawa

NETHERLANDS
- National Aerospace Laboratory (NLR), Library

NEW ZEALAND

Defence Scientific Establishment, Library
RNZAF
Transport Ministry, Airworthiness Branch, Library

Universities
Canterbury
Library
Professor D. Stevenson, Mechanical Engineering

SWEDEN

Aeronautical Research Institute, Library
Swedish National Defense Research Institute (FOA)

SWITZERLAND

Armament Technology and Procurement Group
F+W (Swiss Federal Aircraft Factory)

UNITED KINGDOM

Ministry of Defence, Research, Materials and Collaboration
CAARC, Secretary
CAARC Coordinator Aerodynamics

Royal Aircraft Establishment
Bedford, Library
Pyestock, Director
Farnborough, Dr G. Wood, Materials Department
Commonwealth Air Transport Council Secretariat

Admiralty Research Establishment
Holton Heath, Dr N.J. Wadsorth
St Leonard's Hill, Superintendent
National Industrial Fuel Efficiency Service, Chief Engineer
National Physical Laboratory, Library
National Engineering Laboratory, Library
British Library, Document Supply Centre
CAARC Co-ordinator, Structures
Aircraft Research Association, Library
British Maritime Technology Ltd
NMI Limited (National Maritime Institute), Library
Ruston Gas Turbines Ltd, Technical Director
Fulmer Research Institute Ltd, Research Director
Motor Industry Research Association, Director
Ricardo & Company Engineers (1927) Ltd, Manager
Rolls-Royce Ltd, Aero Division Bristol, Library

British Aerospace
Kingston-upon-Thames, Library
Hatfield-Chester Division, Library
British Hovercraft Corporate Ltd, Library
Short Brothers Ltd, Technical Library

Universities and Colleges
Cranfield Institute of Technology
Library

UNITED STATES OF AMERICA

NASA Scientific and Technical Information Facility
Materials Information, American Society for Metals
The Chemical Abstracts Service

Boeing Company
Mr W.E. Binz
Mr J.C. McMillan
Kentex Research Library
United Technologies Corporation, Library
Lockheed-California Company
Lockheed Missiles and Space Company
Lockheed Georgia
McDonnell Aircraft Company, Library
Nondestructive Testing Information Analysis Center

Spares (20 copies)

Total (139 copies)

AD-A163 00 ...

AL 149
REVISED DECEMBER 87

DEPARTMENT OF DEFENCE

DOCUMENT CONTROL DATA

PAGE CLASSIFICATION
UNCLASSIFIED

PRIVACY MARKING

1a. AR NUMBER	1b. ESTABLISHMENT NUMBER	2. DOCUMENT DATE	3. TASK NUMBER
AR-004-596	ARL-AERO-R-176	FEBRUARY 1988	DST 86/052

4. TITLE
A POTENTIAL THEORY FOR THE STEADY SEPARATED FLOW ABOUT AN AEROFOIL SECTION

5. SECURITY CLASSIFICATION
(PLACE APPROPRIATE CLASSIFICATION IN BOX(S) IE. SECRET (S), CONFIDENTIAL (C), RESTRICTED (R), UNCLASSIFIED (U).)

U	U	U
DOCUMENT	TITLE	ABSTRACT

6. No. PAGES
30

7. No. REFS.
13

8. AUTHOR(S)
TON TRAN-CONG

9. DOWNGRADING/DELIMITING INSTRUCTIONS

10. CORPORATE AUTHOR AND ADDRESS

AERONAUTICAL RESEARCH LABORATORY
P.O. BOX 4331, MELBOURNE VIC. 3001

11. OFFICE/POSITION RESPONSIBLE FOR

SPONSOR ____________

SECURITY ____________

DOWNGRADING ____________

APPROVAL ____________

12. SECONDARY DISTRIBUTION (OF THIS DOCUMENT)
Approved for public release.

OVERSEAS ENQUIRIES OUTSIDE STATED LIMITATIONS SHOULD BE REFERRED THROUGH ASDIS, DEFENCE INFORMATION SERVICES BRANCH, DEPARTMENT OF DEFENCE, CAMPBELL PARK, CANBERRA, ACT 2601

13a. THIS DOCUMENT MAY BE ANNOUNCED IN CATALOGUES AND AWARENESS SERVICES AVAILABLE TO.....
No limitations

13b. CITATION FOR OTHER PURPOSES (IE. CASUAL ANNOUNCEMENT) MAY BE [X] UNRESTRICTED OR [ ] AS FOR 13a.

14. DESCRIPTORS
Airfoils
Flat plate models
Steady flow

15. DRDA SUBJECT CATEGORIES
0051A

16. ABSTRACT
An incompressible potential flow theory is used to determine the steady separated flow about an aerofoil. The theory permits a continuous variation from fully-attached (Joukowski) flow to fully-separated (Helmholtz) flow, with the Kutta condition always satisfied at the trailing edge, and with the position of the separation point as an assignable parameter to determine the flow configuration. The method is also applicable to other flows such as that about a flat plate with a rear free-stream flap.

PAGE CLASSIFICATION
UNCLASSIFIED
PRIVACY MARKING

THIS PAGE IS TO BE USED TO RECORD INFORMATION WHICH IS REQUIRED BY THE ESTABLISHMENT FOR ITS OWN USE BUT WHICH WILL NOT BE ADDED TO THE DISTIS DATA UNLESS SPECIFICALLY REQUESTED.

16. ABSTRACT (CONT.)		
17. IMPRINT AERONAUTICAL RESEARCH LABORATORY, MELBOURNE		
18. DOCUMENT SERIES AND NUMBER AERODYNAMICS REPORT 176	19. COST CODE 525026	20. TYPE OF REPORT AND PERIOD COVERED
21. COMPUTER PROGRAMS USED		
22. ESTABLISHMENT FILE REF. (S)		
23. ADDITIONAL INFORMATION (AS REQUIRED)		